Some results and some challenges in statistical seismology

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Abstract. The Gutenberg-Richter (GR) law is of fundamental importance in statistical seismology. It simply states that, for a given region, the magnitudes of earthquakes follow an exponential probability distribution. As the (scalar) seismic moment is an exponential function of magnitude, when the GR law is expressed in terms of the former variable, it translates into a power-law distribution. The distribution of the seismic moment is of capital interest to evaluate earthquake hazard, in particular regarding the most extreme events; therefore we are going to analyze the tail of the seismic moment law. It is well known that there exist a range of the seismic moment such that the scale-free assumption holds, but several questions are involved. Is the scale-free range bounded? Can we estimate it? Moreover, which is the support of the GR law? We are going to show some recent advances for answering each of these questions.

Mots-clés. Spatial Statistics, Statistical Distributions, Climate and Environment.

Introduction

Physical laws state properties that experimental results show through patterns. On the other hand, probabilistic laws allow us to analyze these patterns and to infer. Statistical seismology is based on two main physical laws: Gutenberg-Richter law and Omori's law. We are going to analyze from a probabilistic point of view the first one.

1 Gutenberg-Richter law as a probabilistic law

The Gutenberg-Richter (GR) law simply states that, for a given region, the magnitudes of earthquakes follow an exponential probability distribution. As the (scalar) seismic moment is an exponential function of magnitude, when the GR law is expressed in terms of the former variable, it translates into a power-law distribution [1], i.e.,

$$f(M) \propto \frac{1}{M^{1+\beta}}, \tag{1}$$

with M seismic moment, $f(M)$ its probability density function, and the exponent $1+\beta$ taking values close to 1.65. This simple description provides rather good fits of available data in many cases [2], with, remarkably, only one free parameter, β. However, a general framework states that the law holds in some range of magnitude. Remarkably, this range is characterized by the scale-free property. Some questions are involved from the statistical point of view: Is the scale-free range bounded? Can we estimate it? Moreover, which is the support of the GR law? We are going to show some recent advances for answering each one of them.

2 On the support of Gutenberg-Richter law

From the statistical point of view, the key point is to estimate an upper limit of a random variable, essentially, bounded or not. The proposed approach for answering is based on extreme value theory, the classification of type of tail produces an answer [3]. In particular, we develop the best scale-free test for inferring [4].

3 On the deviation from power-law

The assumption of non-bounded support for the GR law produces an additional question on the type of tail. Is the tail heavy or not, equivalently, does the GR law hold in all range of scalar seismic moment or not. Many complex systems produce deviation from power-law for extreme events [5]. We analyze the main approaches from the physical point of view for concluding that the exponential decay in the tail has to be considered [6].

4 On the range with scale-free property

The exponential-decay assumption produces an additional question about how to identify the range where the GR law holds, equivalently where the scale-free assumption holds. Several methods produce answers based on the Kolmogorov-Smirnov test [7]. Additionally, we are going to show a method for estimating this range. This methodology is based on log-concavity properties.

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